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Chapter Eight

Rotational Equilibrium and Rotational Dynamics

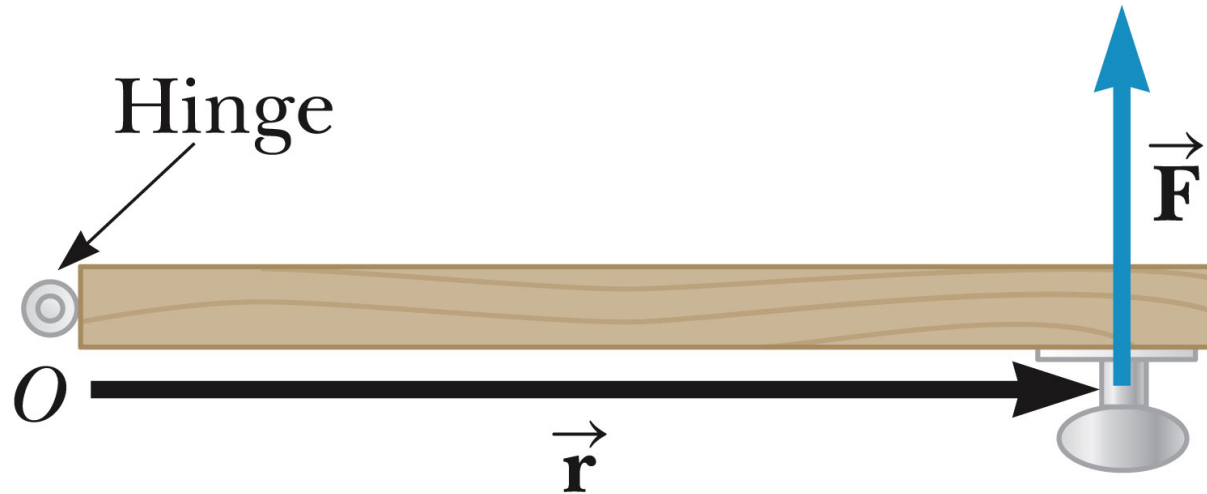
Application of Forces

- The point of application of a force is important
 - This was ignored in treating objects as point particles
- The concepts of rotational equilibrium and rotational dynamics are important in many fields of study
- Angular momentum may be conserved
- Angular momentum may be changed by exerting a torque

Force vs. Torque

- Forces cause accelerations
- Torques cause angular accelerations
- Force and torque are related

Torque



- The door is free to rotate about an axis through O
- There are three factors that determine the effectiveness of the force in opening the door:
 - The *magnitude* of the force
 - The *position* of the application of the force
 - The *angle* at which the force is applied

Torque, cont

- Torque, τ , is the tendency of a force to rotate an object about some axis
 - $\tau = r F$
 - τ is the torque
 - Symbol is the Greek tau
 - r is the length of the position vector
 - F is the force
- SI unit is Newton · meter (N·m)

Direction of Torque

- Torque is a vector quantity
 - The direction is perpendicular to the plane determined by the position vector and the force
 - If the turning tendency of the force is counterclockwise, the torque will be positive
 - If the turning tendency is clockwise, the torque will be negative

Multiple Torques

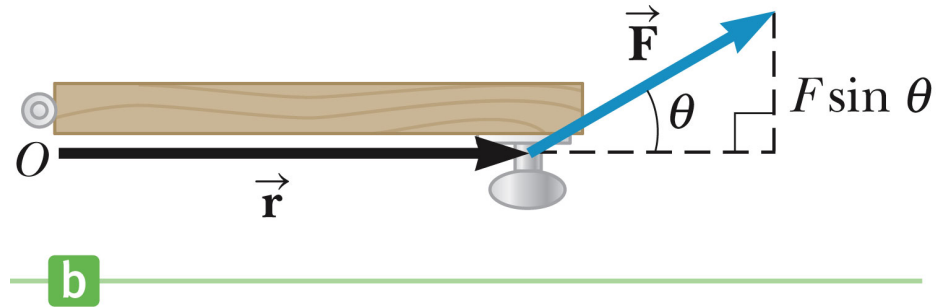
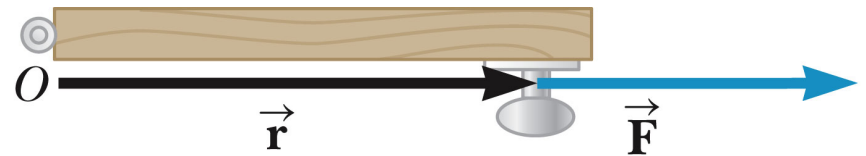
- When two or more torques are acting on an object, the torques are added
 - As vectors
- If the net torque is zero, the object's rate of rotation doesn't change

General Definition of Torque

- The applied force is not always perpendicular to the position vector
- The component of the force *perpendicular* to the object will cause it to rotate

General Definition of Torque, cont

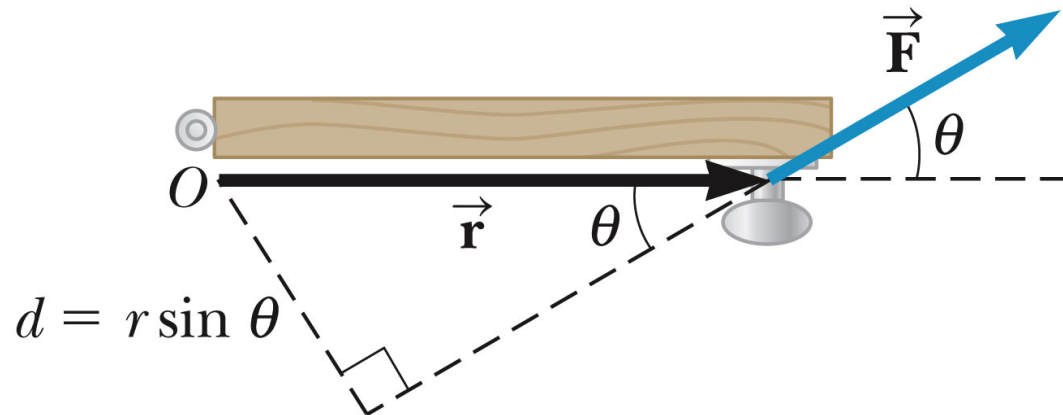
- When the force is parallel to the position vector, no rotation occurs
- When the force is at some angle, the perpendicular component causes the rotation



General Definition of Torque, final

- Taking the angle into account leads to a more general definition of torque:
 - $\tau = r F \sin \theta$
 - r is the position vector
 - F is the force
 - θ is the angle between the force and the position vector

Lever Arm



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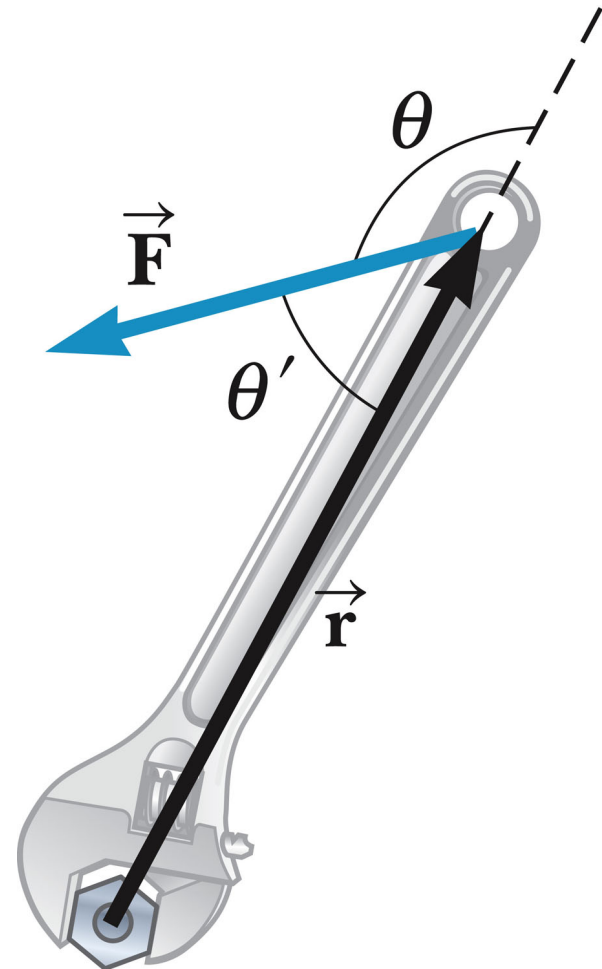
- The *lever arm*, d , is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force
- $d = r \sin \theta$
- This also gives $\tau = r F \sin \theta$

Torque and Axis

- The value of the torque depends on the chosen axis of rotation
- Torques can be computed around any axis
 - There doesn't have to be a physical rotation axis present
- Once a point is chosen, it must be used consistently throughout a given problem

Right Hand Rule

- Point the fingers in the direction of the position vector
- Curl the fingers toward the force vector
- The thumb points in the direction of the torque



Net Torque

- The net torque is the sum of all the torques produced by all the forces
 - Remember to account for the direction of the tendency for rotation
 - Counterclockwise torques are positive
 - Clockwise torques are negative

Torque and Equilibrium

- First Condition of Equilibrium

- The net external force must be zero

$$\Sigma \vec{\mathbf{F}} = 0 \text{ or } \Sigma \vec{\mathbf{F}}_x = 0 \text{ and } \Sigma \vec{\mathbf{F}}_y = 0$$

- This is a statement of translational equilibrium

- The Second Condition of Equilibrium states

- The net external torque must be zero

$$\Sigma \vec{\tau} = 0$$

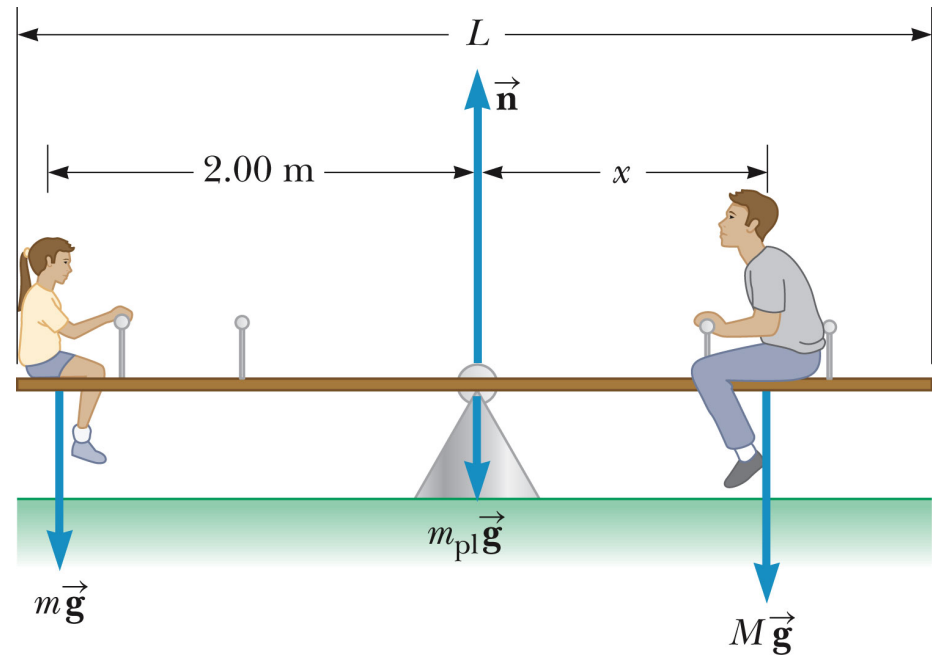
- This is a statement of rotational equilibrium

Selecting an Axis

- It's usually best to choose an axis that will make at least one torque equal to zero
 - This will simplify the torque equation

Equilibrium Example

- The woman, mass m , sits on the left end of the see-saw
- The man, mass M , sits where the see-saw will be balanced
- Apply the Second Condition of Equilibrium and solve for the unknown distance, x

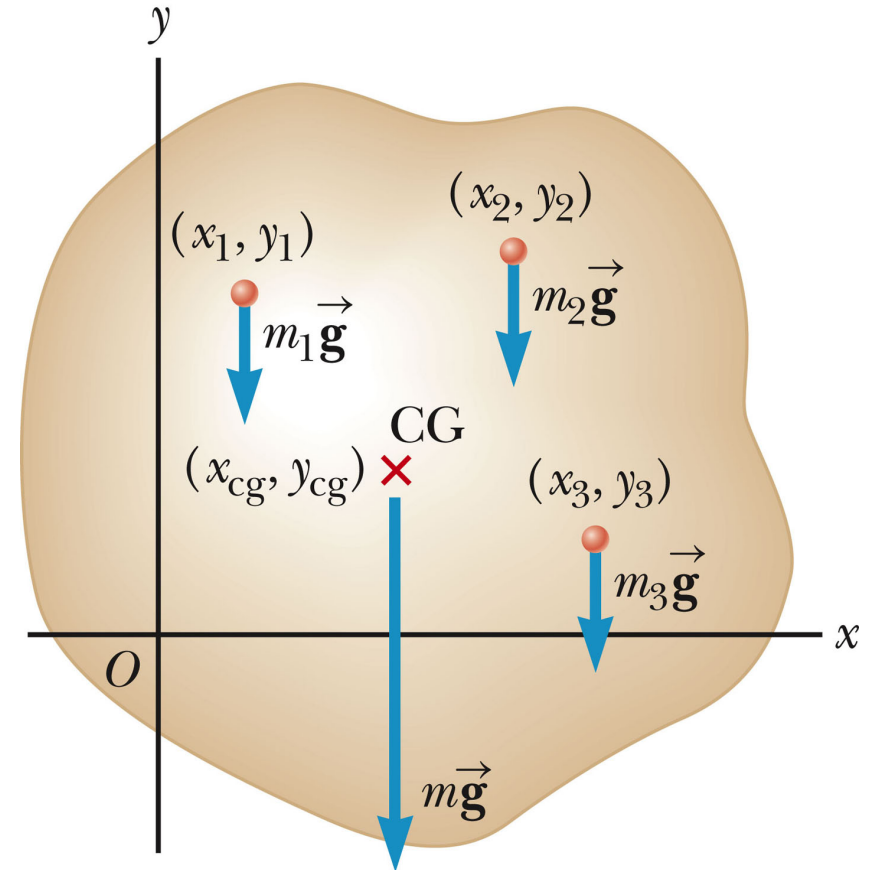


Center of Gravity

- The force of gravity acting on an object must be considered
- In finding the torque produced by the force of gravity, all of the weight of the object can be considered to be concentrated at a single point

Calculating the Center of Gravity

- The object is divided up into a large number of very small particles of weight (mg)
- Each particle will have a set of coordinates indicating its location (x,y)



Calculating the Center of Gravity, cont.

- We assume the object is free to rotate about its center
- The torque produced by each particle about the axis of rotation is equal to its weight times its lever arm
 - For example, $\tau_1 = m_1 g x_1$

Calculating the Center of Gravity, cont.

- We wish to locate the point of application of the *single force* whose magnitude is equal to the weight of the object, and whose effect on the rotation is the same as all the individual particles.
- This point is called the *center of gravity* of the object

Coordinates of the Center of Gravity

- The coordinates of the center of gravity can be found from the sum of the torques acting on the individual particles being set equal to the torque produced by the weight of the object

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} \text{ and } y_{cg} = \frac{\sum m_i y_i}{\sum m_i} \text{ and } z_{cg} = \frac{\sum m_i z_i}{\sum m_i}$$

Center of Gravity and Center of Mass

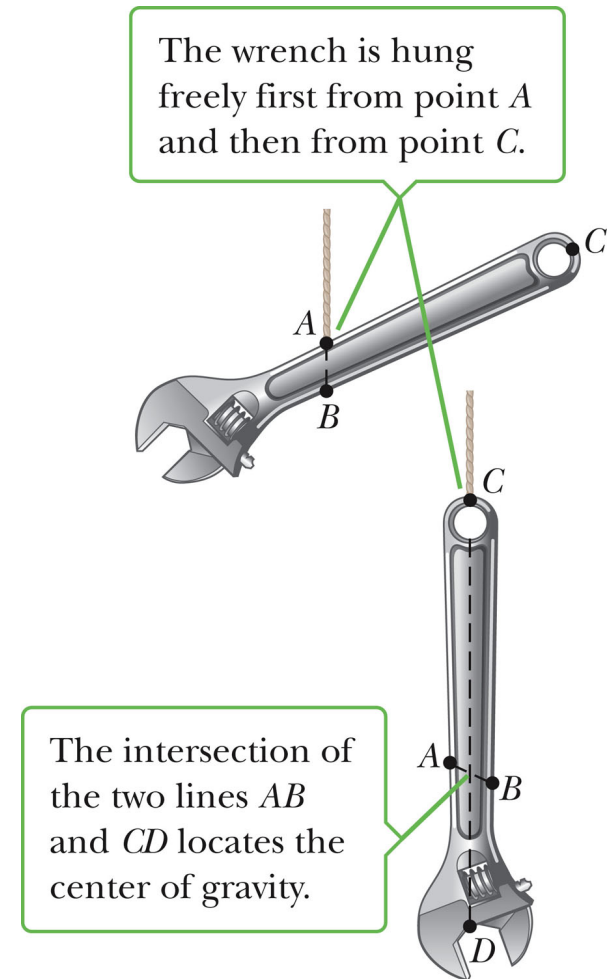
- The three equations giving the coordinates of the center of gravity of an object are identical to the equations giving the coordinates of the *center of mass* of the object
- The center of gravity and the center of mass of the object are the same if the value of g does not vary significantly over the object

Center of Gravity of a Uniform Object

- The center of gravity of a homogenous, symmetric body must lie on the axis of symmetry
- Often, the center of gravity of such an object is the *geometric* center of the object

Experimentally Determining the Center of Gravity

- The wrench is hung freely from two different pivots
- The intersection of the lines indicates the center of gravity
- A rigid object can be balanced by a single force equal in magnitude to its weight as long as the force is acting upward through the object's center of gravity



Notes About Equilibrium

- A zero net torque does not mean the absence of rotational motion
 - An object that rotates at uniform angular velocity can be under the influence of a zero net torque
 - This is analogous to the translational situation where a zero net force does not mean the object is not in motion

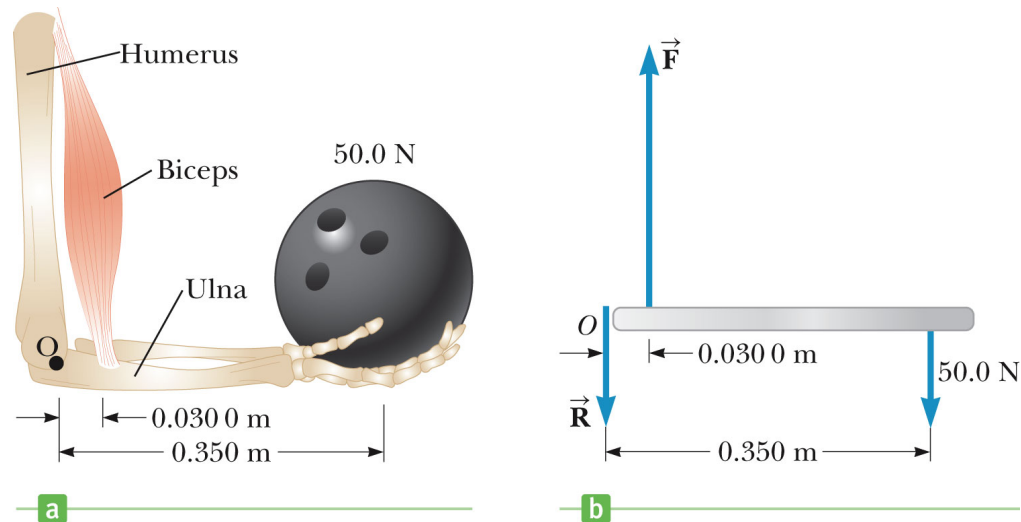
Solving Equilibrium Problems

- Diagram the system
 - Include coordinates and choose a convenient rotation axis
- Draw a free body diagram showing all the external forces acting on the object
 - For systems containing more than one object, draw a separate free body diagram for each object

Problem Solving, cont.

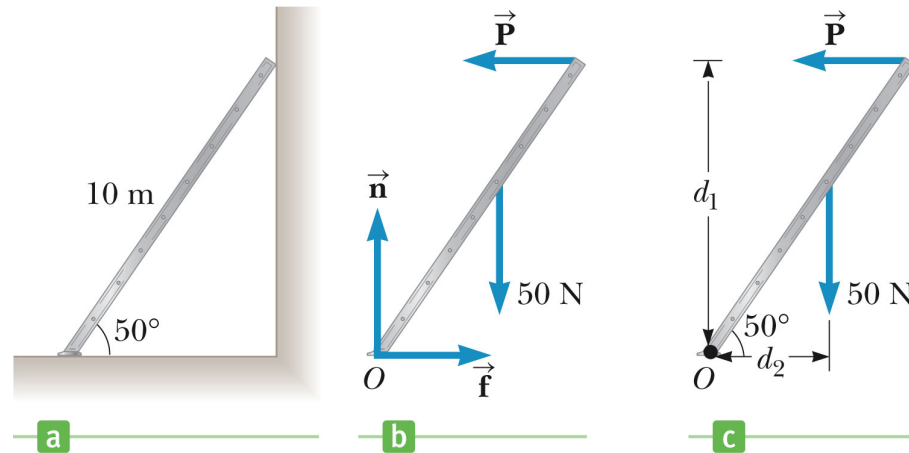
- Apply the Second Condition of Equilibrium
 - This will yield a single equation, often with one unknown which can be solved immediately
- Apply the First Condition of Equilibrium
 - This will give you two more equations
- Solve the resulting simultaneous equations for all of the unknowns
 - Solving by substitution is generally easiest

Example of a Free Body Diagram (Forearm)



- Isolate the object to be analyzed
- Draw the free body diagram for that object
 - Include all the external forces acting on the object

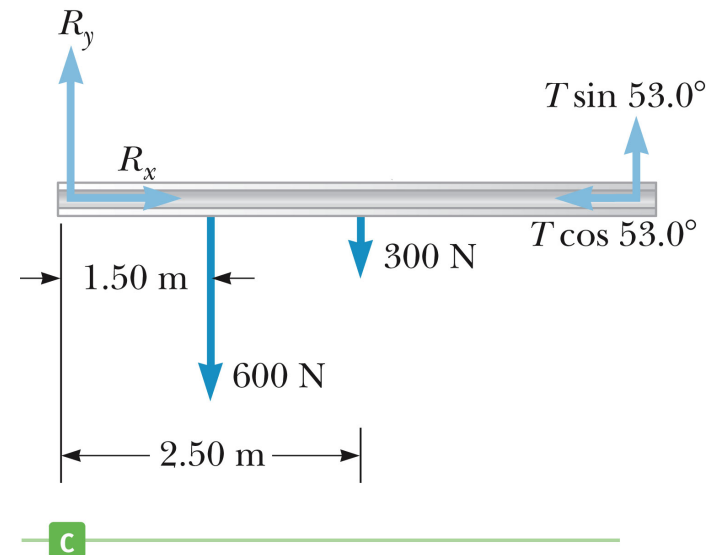
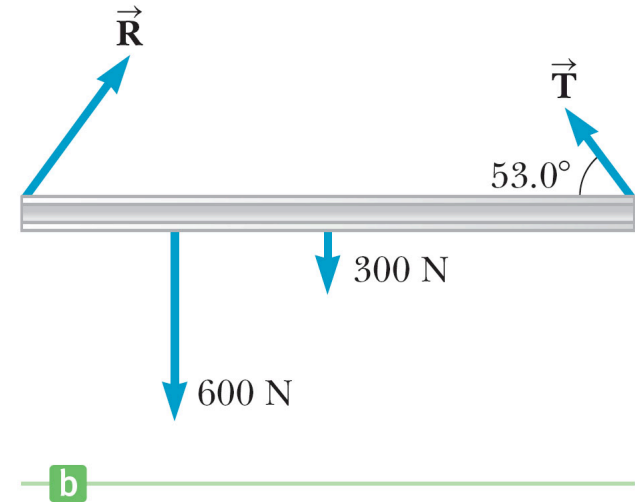
Example of a Free Body Diagram (Ladder)



- The free body diagram shows the normal force and the force of static friction acting on the ladder at the ground
- The last diagram shows the lever arms for the forces

Example of a Free Body Diagram (Beam)

- The free body diagram includes the directions of the forces
- The weights act through the centers of gravity of their objects



Torque and Angular Acceleration

- When a rigid object is subject to a net torque ($\Sigma\tau \neq 0$), it undergoes an angular acceleration
- The angular acceleration is directly proportional to the net torque
 - The relationship is analogous to $\Sigma F = ma$
 - Newton's Second Law

Moment of Inertia

- The angular acceleration is inversely proportional to the analogy of the mass in a rotating system
- This mass analog is called the *moment of inertia*, I , of the object

$$I \equiv \sum mr^2$$

– SI units are kg m^2

Newton's Second Law for a Rotating Object

$$\Sigma \tau = I\alpha$$

- The angular acceleration is directly proportional to the net torque
- The angular acceleration is inversely proportional to the moment of inertia of the object

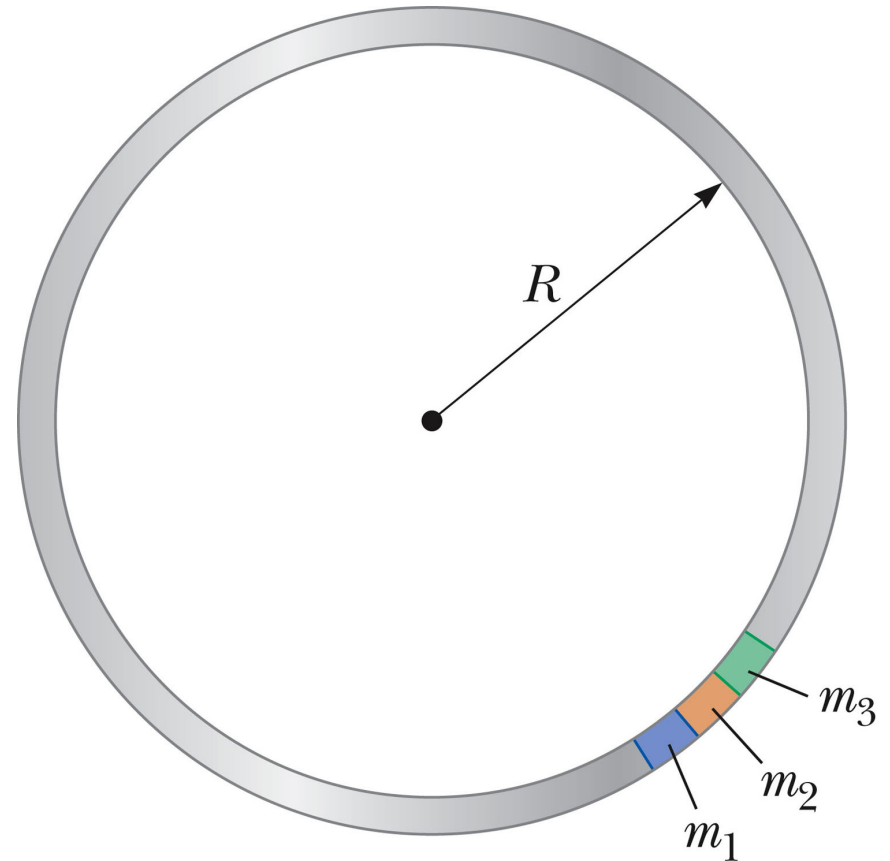
More About Moment of Inertia

- There is a major difference between moment of inertia and mass: the moment of inertia depends on the quantity of matter and its distribution in the rigid object
- The moment of inertia also depends upon the location of the axis of rotation

Moment of Inertia of a Uniform Ring

- Imagine the hoop is divided into a number of small segments, $m_1 \dots$
- These segments are equidistant from the axis

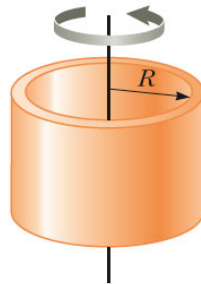
$$I = \sum m_i r_i^2 = MR^2$$



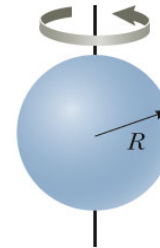
Other Moments of Inertia

Table 8.1 Moments of Inertia for Various Rigid Objects of Uniform Composition

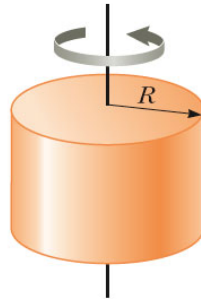
Hoop or thin
cylindrical shell
 $I = MR^2$



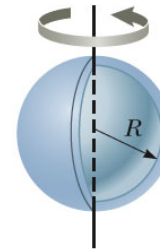
Solid sphere
 $I = \frac{2}{5}MR^2$



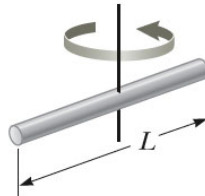
Solid cylinder
or disk
 $I = \frac{1}{2}MR^2$



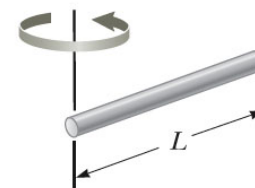
Thin spherical
shell
 $I = \frac{2}{3}MR^2$



Long, thin rod
with rotation axis
through center
 $I = \frac{1}{12}ML^2$

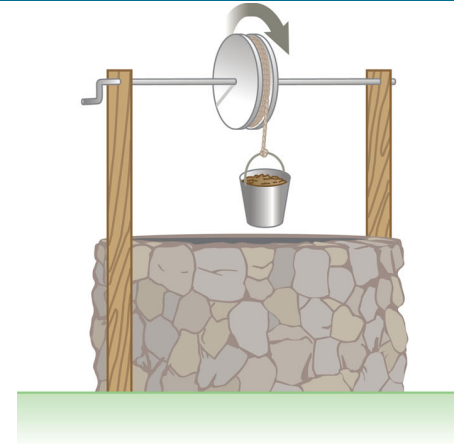


Long, thin
rod with
rotation axis
through end
 $I = \frac{1}{3}ML^2$



Example, Newton's Second Law for Rotation

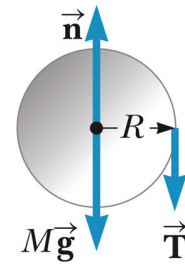
- Draw free body diagrams of each object
- Only the cylinder is rotating, so apply $\Sigma\tau = I\alpha$
- The bucket is falling, but not rotating, so apply $\Sigma F = ma$
- Remember that $a = \alpha r$ and solve the resulting equations



a



b



c

Rotational Kinetic Energy

- An object rotating about some axis with an angular speed, ω , has rotational kinetic energy
 $KE_r = \frac{1}{2}I\omega^2$
- Energy concepts can be useful for simplifying the analysis of rotational motion

Conservation of Energy

- Conservation of Mechanical Energy

$$(KE_t + KE_r + PE_g)_i = (KE_t + KE_r + PE_g)_f$$

- Remember, this is for conservative forces, no dissipative forces such as friction can be present
- Potential energies of any other conservative forces could be added

Work-Energy in a Rotating System

- In the case where there are dissipative forces such as friction, use the generalized Work-Energy Theorem instead of Conservation of Energy
- $W_{nc} = \Delta KE_t + \Delta KE_r + \Delta PE$

Problem Solving Hints for Energy Methods

- Choose two points of interest
 - One where all the necessary information is given
 - The other where information is desired
- Identify the conservative and nonconservative forces

Problem Solving Hints for Energy Methods, cont

- Write the general equation for the Work-Energy theorem if there are nonconservative forces
 - Use Conservation of Energy if there are no nonconservative forces
- Use $v = r \omega$ to eliminate either ω or v from the equation
- Solve for the unknown

Angular Momentum

- Similarly to the relationship between force and momentum in a linear system, we can show the relationship between torque and angular momentum
- Angular momentum is defined as

- $L = I \omega$

- and $\Sigma \tau = \frac{\Delta L}{\Delta t}$

Angular Momentum, cont

- If the net torque is zero, the angular momentum remains constant
- *Conservation of Angular Momentum* states: Let L_i and L_f be the angular momenta of a system at two different times, and suppose there is no net external torque so $\Sigma \tau = 0$, then angular momentum is said to be conserved

Conservation of Angular Momentum

- Mathematically, when

$$\Sigma \tau = 0, L_i = L_f \text{ or } I_i \omega_i = I_f \omega_f$$

- Applies to macroscopic objects as well as atoms and molecules

Conservation of Angular Momentum, Example

- With hands and feet drawn closer to the body, the skater's angular speed increases
 - L is conserved, I decreases, ω increases

By pulling in his arms and legs, he reduces his moment of inertia and increases his angular speed (rate of spin).



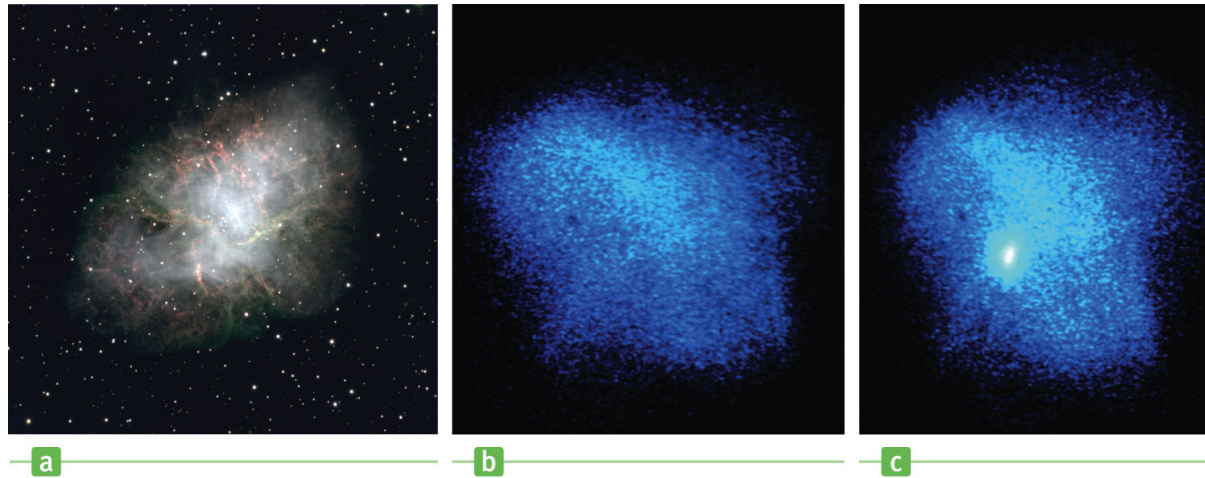
Conservation of Angular Momentum, Example, cont

- Coming out of the spin, arms and legs are extended and rotation is slowed
 - L is conserved, I increases, ω decreases

Upon landing, extending his arms and legs increases his moment of inertia and helps slow his spin.



Conservation of Angular Momentum, Astronomy Example



- Crab Nebula, result of supernova
- Center is a neutron star
 - As the star's moment of inertia decreases, its rotational speed increases