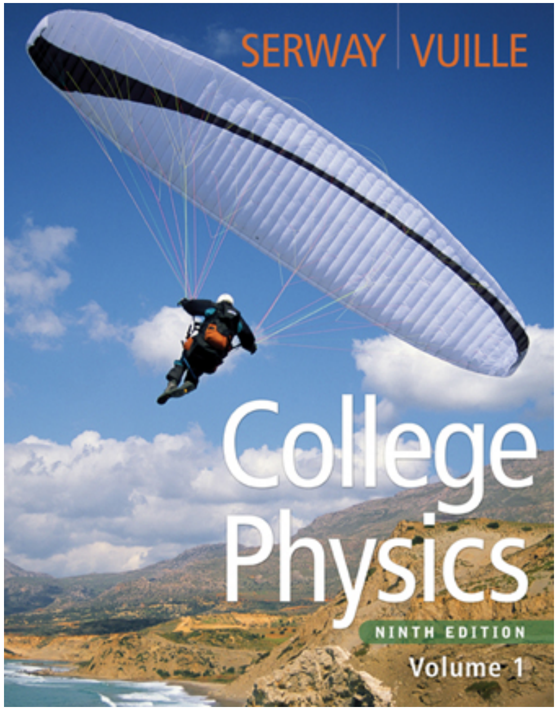

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Raymond A. Serway
Chris Vuille

Chapter Six

Momentum and Collisions

Collisions

- Conservation of momentum allows complex collision problems to be solved without knowing about the forces involved
- Information about the average force can be derived

Momentum

- The linear momentum $\vec{\mathbf{p}}$ of an object of mass *m* moving with a velocity $\vec{\mathbf{v}}$ is defined as the product of the mass and the velocity
  - $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
  - SI Units are kg m / s
  - Vector quantity, the direction of the momentum is the same as the velocity's

More About Momentum

- Momentum components
  - $p_x = m\, v_x$ *and* $p_y = m\, v_y$
    - Applies to two-dimensional motion
- Momentum is related to kinetic energy
  - $KE = \dfrac{p^2}{2m}$

Impulse

- In order to *change* the momentum of an object, a force must be applied
- The time rate of change of momentum of an object is equal to the net force acting on it
  - $$\frac{\Delta \vec{\mathbf{p}}}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t} = \vec{\mathbf{F}}_{net}$$
  - Gives an alternative statement of Newton's second law
  - Also valid when the forces are not constant

Impulse cont.

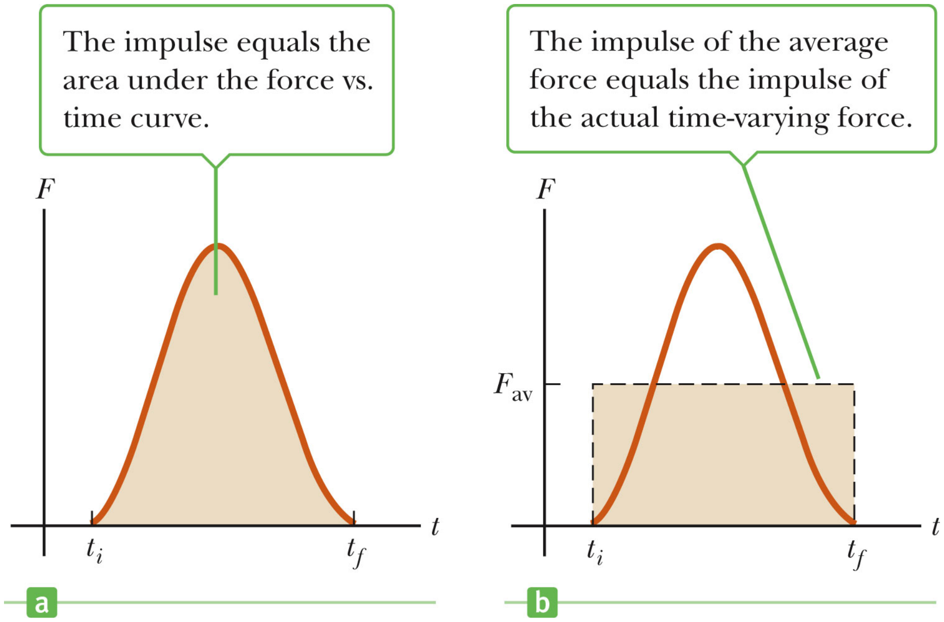
- When a single, constant force acts on the object, there is an **impulse** delivered to the object
  - $\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t$
  - $\vec{\mathbf{I}}$ is defined as the *impulse*
  - Vector quantity, the direction is the same as the direction of the force
  - SI unit of impulse: $kg \cdot m / s$

Impulse-Momentum Theorem

- The theorem states that the impulse acting on the object is equal to the change in momentum of the object
  - $\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}} = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$
  - If the force is not constant, use the *average force* applied

Average Force in Impulse

- The average force can be thought of as the constant force that would give the same impulse to the object in the time interval as the actual time-varying force gives in the interval



Average Force cont.

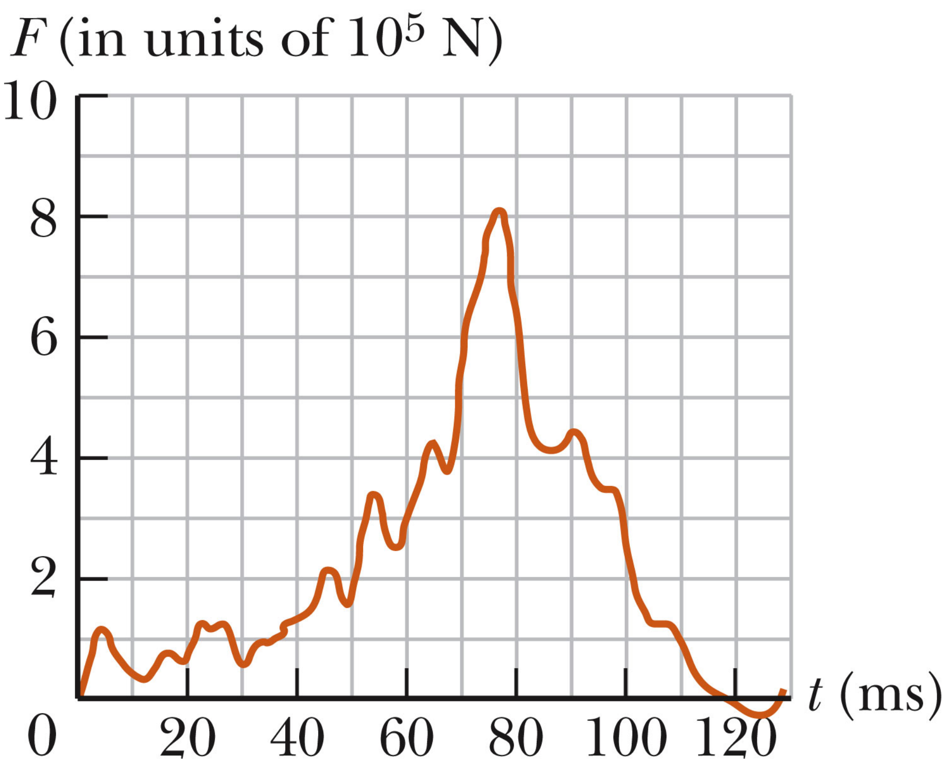
- The impulse imparted by a force during the time interval Δt is equal to the area under the force-time graph from the beginning to the end of the time interval
- Or, the impulse is equal to the average force multiplied by the time interval, $\vec{\mathbf{F}}_{av}\Delta t = \Delta\vec{\mathbf{p}}$

Impulse Applied to Auto Collisions

- The most important factor is the collision time, or the time it takes the person to come to a rest
  - Increasing this time will reduce the chance of dying in a car crash
- Ways to increase the time
  - Seat belts
  - Air bags

Typical Collision Values

- For a 75 kg person traveling at 27 m/s (60.0 mph) and coming to stop in 0.010 s
- F = -2.0×10^5 N
- a = 280 *g*
- Almost certainly fatal



Seat Belts

- Seat belts
  - Restrain people so it takes more time for them to stop
  - New time is about 0.15 seconds
  - New force is about 9.8 kN
  - About one order of magnitude below the values for an unprotected collision

Air Bags

- The air bag increases the time of the collision
- It will also absorb some of the energy from the body
- It will spread out the area of contact
  - Decreases the pressure
  - Helps prevent penetration wounds

Conservation of Momentum

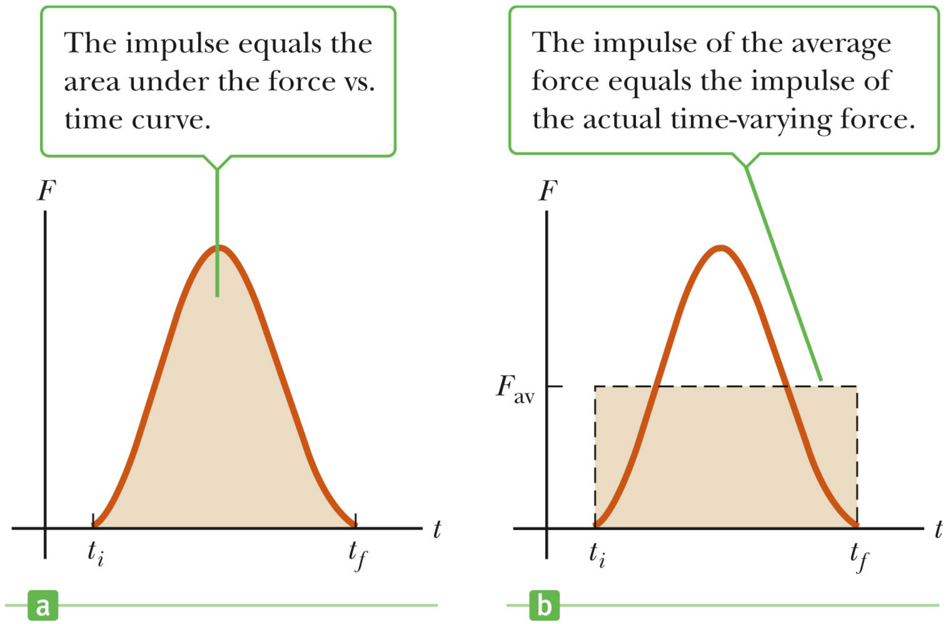
- Momentum in an isolated system in which a collision occurs is conserved
  - A collision may be the result of physical contact between two objects
  - "Contact" may also arise from the electrostatic interactions of the electrons in the surface atoms of the bodies
  - An isolated system will have not external forces

Conservation of Momentum, cont

- The principle of conservation of momentum states when no external forces act on a system consisting of two objects that collide with each other, the total momentum of the system remains constant in time
  - Specifically, the total momentum before the collision will equal the total momentum after the collision

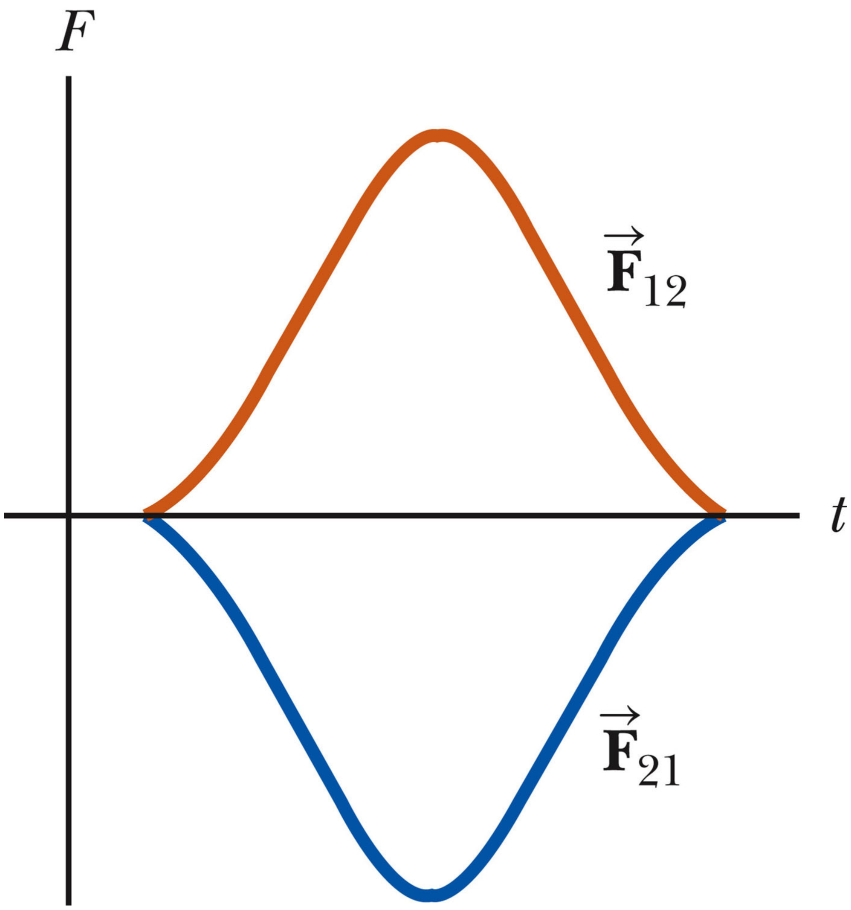
Conservation of Momentum, Example

- The momentum of each object will change
- The total momentum of the system remains constant



Forces in a Collision

- The force with which object 1 acts on object 2 is equal and opposite to the force with which object 2 acts on object 1
- Impulses are also equal and opposite



Conservation of Momentum, cont.

- Mathematically:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

  - Momentum is conserved for the *system* of objects
  - The system includes all the objects interacting with each other
  - Assumes only internal forces are acting during the collision
  - Can be generalized to any number of objects

Types of Collisions

- Momentum is conserved in any collision
- Inelastic collisions
  - Kinetic energy is not conserved
    - Some of the kinetic energy is converted into other types of energy such as heat, sound, work to permanently deform an object
  - Perfectly inelastic collisions occur when the objects stick together
    - Not all of the KE is necessarily lost

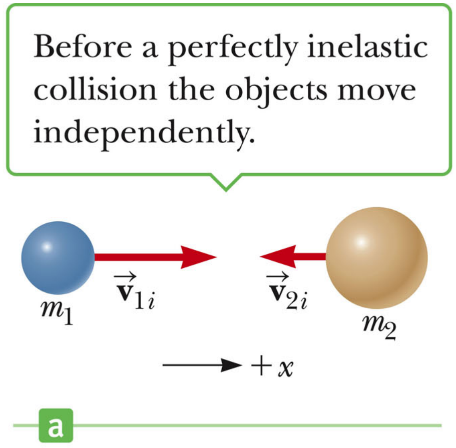
More Types of Collisions

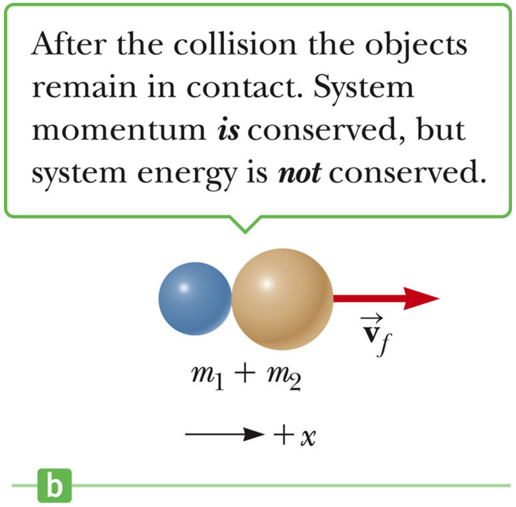
- Elastic collision
  - Both momentum and kinetic energy are conserved
- Actual collisions
  - Most collisions fall between elastic and perfectly inelastic collisions

Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision
- Conservation of momentum becomes

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$





Some General Notes About Collisions

- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs

More About Elastic Collisions

- Both momentum and kinetic energy are conserved
- Typically have two unknowns

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

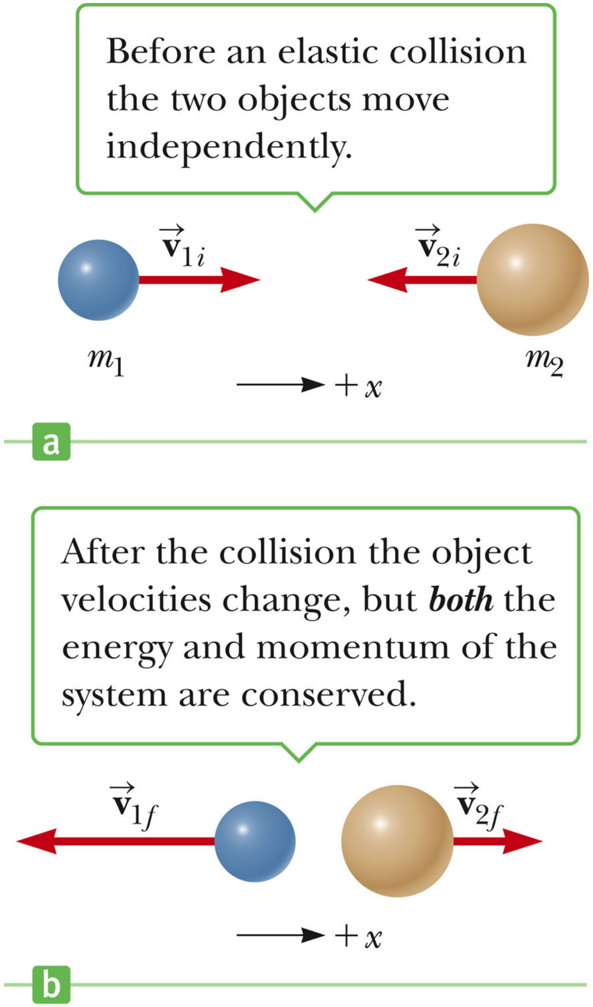
$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

- Solve the equations simultaneously

Elastic Collisions, cont.

- A simpler equation can be used in place of the KE equation

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$



Summary of Types of Collisions

- In an elastic collision, both momentum and kinetic energy are conserved
- In an inelastic collision, momentum is conserved but kinetic energy is not
- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same

Problem Solving for One -Dimensional Collisions

- **Coordinates:** Set up a coordinate axis and define the velocities with respect to this axis
  - It is convenient to make your axis coincide with one of the initial velocities
- **Diagram:** In your sketch, draw all the velocity vectors and label the velocities and the masses

Problem Solving for One -Dimensional Collisions, 2

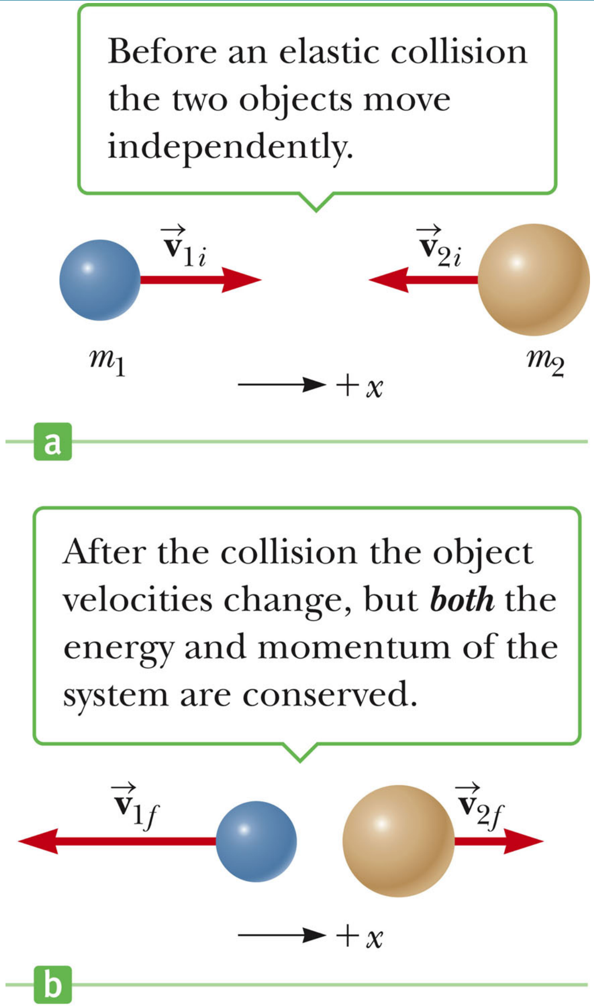
- **Conservation of Momentum:** Write a general expression for the total momentum of the system *before* and *after* the collision
  - Equate the two total momentum expressions
  - Fill in the known values

Problem Solving for One -Dimensional Collisions, 3

- **Conservation of Energy:** If the collision is elastic, write a second equation for conservation of KE, or the alternative equation
  - This only applies to perfectly elastic collisions
- **Solve:** the resulting equations simultaneously

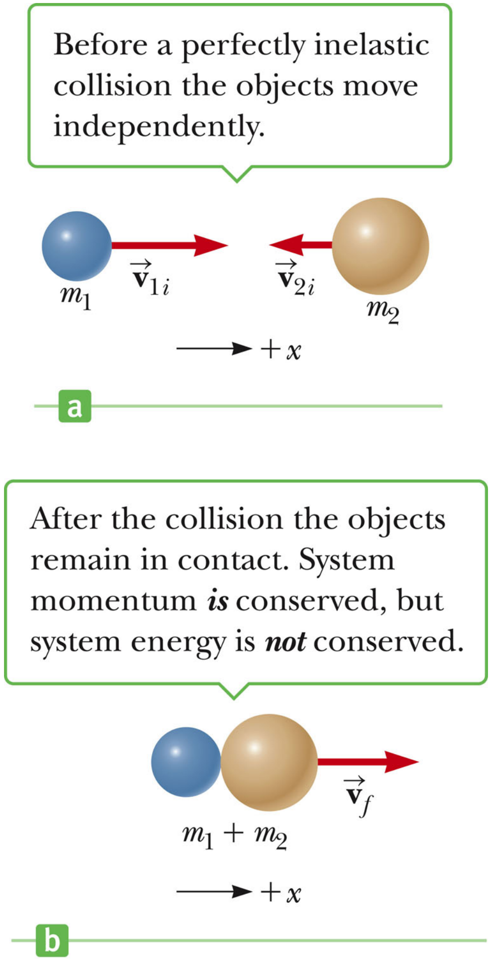
Sketches for Collision Problems

- Draw "before" and "after" sketches
- Label each object
  - Include the direction of velocity
  - Keep track of subscripts



Sketches for Perfectly Inelastic Collisions

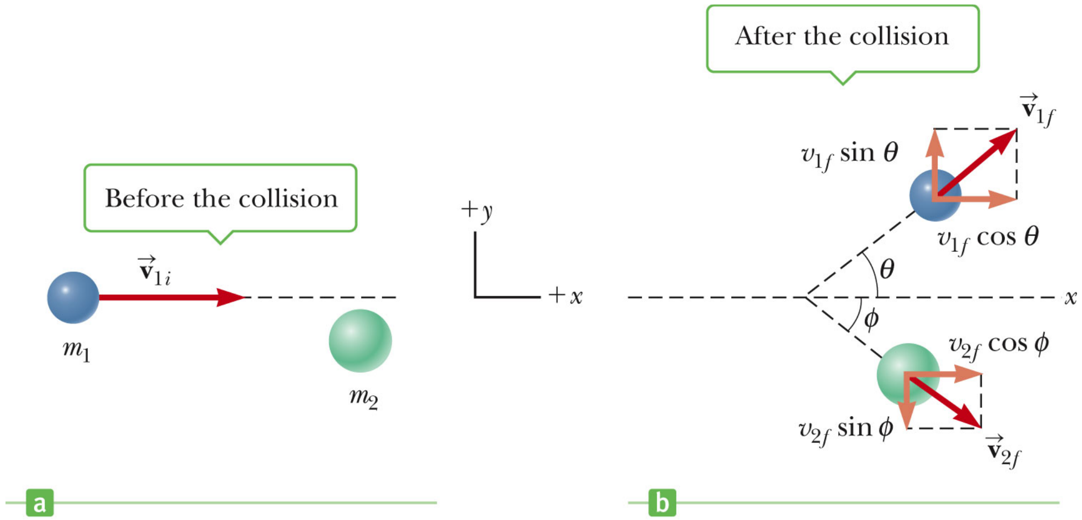
- The objects stick together
- Include all the velocity directions
- The "after" collision combines the masses
- Both move with the same velocity



Glancing Collisions

- For a general collision of two objects in three-dimensional space, the conservation of momentum principle implies that the *total momentum of the system in each direction is conserved*
  - $m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$ and
    $m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$
  - Use subscripts for identifying the object, initial and final velocities, and components

Glancing Collisions – Example



- The "after" velocities have x and y components
- Momentum is conserved in the x direction and in the y direction
- Apply conservation of momentum separately to each direction

Problem Solving for Two-Dimensional Collisions

- **Coordinates:** Set up both coordinate axes and define your velocities with respect to these axes
  - It is convenient to choose the x- or y- axis to coincide with one of the initial velocities
- **Diagram:** In your sketch, draw and label all the velocities and masses

Problem Solving for Two-Dimensional Collisions, 2

- **Conservation of Momentum:** Write expressions for the x and y components of the momentum of each object before and after the collision
  - Write expressions for the total momentum before and after the collision in the x-direction and in the y-direction

Problem Solving for Two-Dimensional Collisions, 3

- **Conservation of Energy:** If the collision is perfectly elastic, write an expression for the total energy before and after the collision
  - Equate the two expressions
  - Fill in the known values
  - Solve the quadratic equations
    - Can't be simplified
  - Remember to skip this step if the collision is not perfectly elastic

Problem Solving for Two-Dimensional Collisions, 4

- **Solve** for the unknown quantities
  - Solve the equations simultaneously
  - There will be two equations for inelastic collisions
  - There will be three equations for elastic collisions

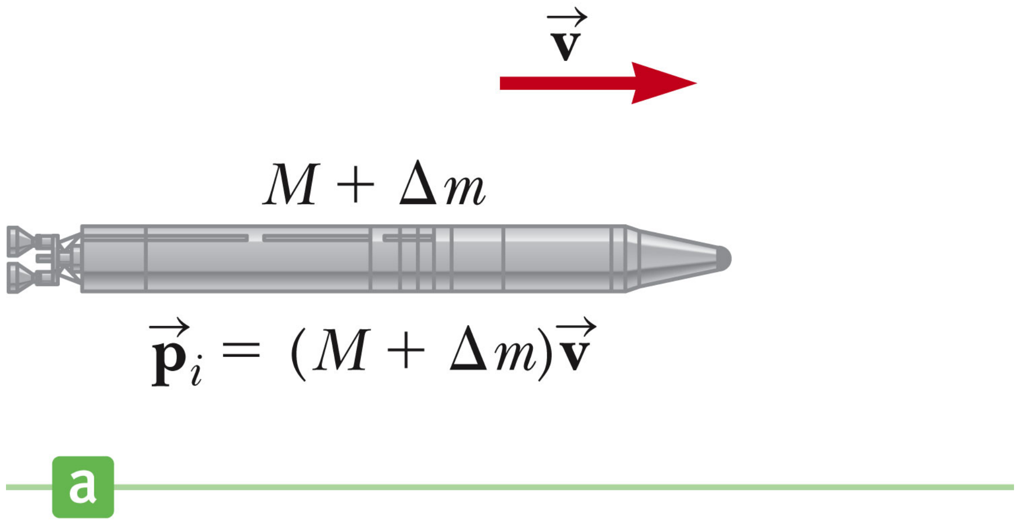
Rocket Propulsion

- The operation of a rocket depends on the law of conservation of momentum as applied to a system, where the system is the rocket plus its ejected fuel
  - This is different than propulsion on the earth where two objects exert forces on each other
    - Road on car
    - Train on track

Rocket Propulsion, 2

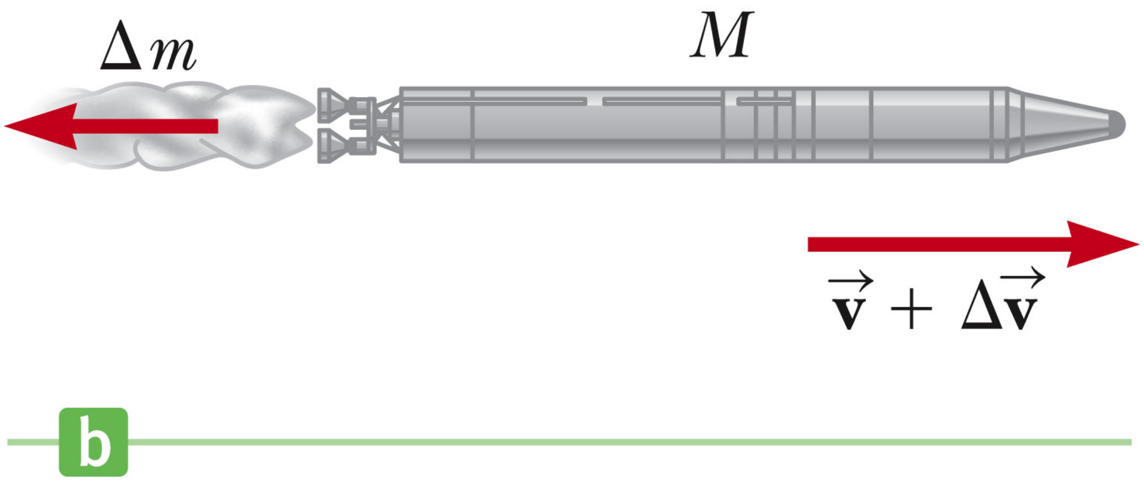
- The rocket is accelerated as a result of the thrust of the exhaust gases
- This represents the inverse of an inelastic collision
  - Momentum is conserved
  - Kinetic Energy is increased (at the expense of the stored energy of the rocket fuel)

Rocket Propulsion, 3



- The initial mass of the rocket is M + Δm
  - M is the mass of the rocket
  - Δm is the mass of the fuel about to be burned
- The initial velocity of the rocket is $\vec{\mathbf{v}}$

Rocket Propulsion



- The rocket's mass is M
- The mass of the fuel, Δm, has been ejected
- The rocket's speed has increased to $\vec{\mathbf{v}} + \Delta\vec{\mathbf{v}}$

Rocket Propulsion, final

- The basic equation for rocket propulsion is:

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

  - M_i is the initial mass of the rocket plus fuel
  - M_f is the final mass of the rocket plus any remaining fuel
  - The speed of the rocket is proportional to the exhaust speed
  - For best results, the exhaust speed should be as high as possible
  - Typical exhaust speeds are several kilometers per second

Thrust of a Rocket

- The thrust is the force exerted on the rocket by the ejected exhaust gases
- The instantaneous thrust is given by

$$Ma = M\frac{\Delta v}{\Delta t} = \left| v_e \frac{\Delta M}{\Delta t} \right|$$

  – The thrust increases as the exhaust speed increases and as the burn rate ($\Delta M/\Delta t$) increases